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Distributed Detection of Weak Signals from Multiple Sensors with Correlated Observations

by

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DISTRIBUTED DETECTION OF WEAK SIGNALS FROM

MULTIPLE SENSORS WITH CORRELATED OBSERVATIONS

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ABSTRACT

We address two problems of distributed detection of a weak signal from dependent observations. In the first problem, two detectors must decide on the basis of their observations whether a weak signal is present or not. The observations of the two detectors consist of a common weak signal disturbed by two independent additive m-dependent or hi-mixing noise processes. Fixed-sample-size (block) detection is employed. The decisions are coupled through a common cost function, which consists of the sum of the error probabilities under the two hypotheses. In the second problem, the observations of each individual detector still consist of a common weak signal disturbed by an additive m-dependent or hi-mixing noise process, but the noise processes of the two detectors are now correlated. The cost function has a structure similar to that of the first problem.

In both cases, the detectors employ suboptimal decision tests based on memoryless nonlinearities. Since the signal is weak, large sample sizes are necessary to guarantee high quality tests and the asymptotic performance is of interest. To determine the optimal nonlinearities for the two detectors, we identify new performance measures based on two-dimensional Chernoff bounds, which correspond to the asymptotic relative efficiency (ARE) used for single-detector problems, and whose maximization implies the minimization of the aforementioned average cost function. This optimization results in integral equations whose solution provides the optimal nonlinearities. Numerical results based on simulation of the performance of the proposed two-sensor schemes are provided to support the analysis.

I. Introduction

Decentralized detection presents several original research problems (see [1] and [2]) because classical optimal detection theory cannot be applied directly to practical distributed sensors systems. To make the analysis tractable, the models employed assume independent data across time and/or sensors, while in reality, the observations are always dependent. Indeed, as the locations of the sensors are close geographically in many practical situations, the noise is correlated and the observations (signal in noise) are dependent across detectors. In this paper, the distributed detection of a weak signal from dependent data is addressed; we consider that two detectors working together have to decide which hypothesis is true H_0 (signal absent) or H_1 (signal present). The signal is a constant weak signal in additive noise; we also assume that there is no communication between the two detectors, although they are coupled through the associated cost function.

A stationary sequence $\{Y_k\}_{k=1}^{\infty}$ is said to be ϕ—mixing, if for $i \leq 1$, $j \leq 1$ and $B_1 \in \mathcal{F}_{i+j}^{\infty}$ there is a real sequence $\{\phi_k\}_{k=0}^{\infty}$ such that

$$\sup_{B_2 \in \mathcal{F}_1^i} |P(B_1 \cap B_2) - P(B_1)P(B_2)| \leq \phi_j P(B_1)$$

and

$$\lim_{k \to \infty} \phi_k = 0,$$

where $\mathcal{F}_{i+j}^{\infty}$ is the σ-field generated by $\{Y_k\}_{k=i+j}^{\infty}$ and $\mathcal{F}_1^i$ is the σ-field generated by $\{Y_k\}_{k=1}^{i}$.

Here the model of dependent noise is described by a stationary ϕ—mixing dependence and each detector is assumed to employ a memoryless nonlinearity. Memoryless nonlinearities have been successfully applied in single-sensor detection problems (see [3] and [4]). The performance measure used for this case is the average cost of the decisions of the two

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detectors. We use the central limit theorem for dependent random sequences (see [5]) and develop a two-dimensional Chernoff bound for the error probabilities of the two-sensor detection problem. Then suitable exponents are chosen on the basis of these bounds as the objective functions for the optimal design of distributed system. We derive the optimal nonlinearities according to these objective functions.

The remainder of this paper is organized as follows: in Section II, we cite the central limit theorem for stationary dependent random sequences and define the expected cost. Then the two-dimensional Chernoff bound is derived. In Section III, the case of dependent data across time– (that is the observations of each sensor are dependent sequences but they are mutually independent for the different sensors when conditioned on H_i ($i = 0, 1$) being true)– is considered and optimal nonlinearities are derived by solving two decoupled linear integral equations. In Section IV, we consider a simple case of dependent data across time and detectors. In Section V, simulation results are presented for both cases described in Sections III and IV for a first-order Markov dependence model and Gaussian as well as Rayleigh multivariate joint probability density functions. of the noise process. Finally, in Section VI conclusions are drawn.

II. Preliminaries

IIA. The Weak Signal Model and the Central Limit Theorem for Dependent Data

We consider the following observation model of a weak signal for two-detector detection

$$H_0^{(k)} \quad : \quad X_i^{(k)} = N_i^{(k)}$$

$$H_1^{(k)} \quad : \quad X_i^{(k)} = \theta + N_i^{(k)}, \quad i = 1, \cdots, n; \ k = 1, 2 \tag{1}$$

where $\theta = M/\sqrt{n}$ is the weak signal, $\{N_i^{(k)}, i = 1, \cdots, n; k = 1, 2\}$ a stationary ϕ-mixing noise sequence with identical univariate distribution f_k for $k = 1, 2$, and M a known positive constant. We attempt an asymptotic analysis for large sample sizes. The detection structure employed by each sensor is the the same as that of [3], namely the one with a memoryless nonlinearity $g_k(\cdot)(k = 1, 2)$. The test statistic for each sensor has the form

$$T_k(\underline{X}^{(k)}) = \sum_{i=1}^{n} g_k(X_i^{(k)}), \quad k = 1, 2 \tag{2}$$

The choice of thresholds is studied in the next two sections. Here, we cite the central limit theorem for ϕ-mixing processes see [5].

Theorem 1: Suppose that $\{Y_j\}_{j=1}^{\infty}$ is a stationary ϕ-mixing sequence with $\sum_{j=1}^{\infty} \phi_j^{\frac{1}{2}} < \infty$ and that g is a measurable function satisfying

$$E_0[g(Y_1)] = \mu_0, \quad var_0[g(Y_1)] < \infty$$

Then the series

$$\sigma_0^2(g) = var_0[g(Y_1)] + 2 \sum_{j=1}^{\infty} cov_0[g(Y_1)g(Y_{j+1})] \tag{3}$$

converges absolutely. Furthermore, if $\sigma_0^2 > 0$,

$$\frac{1}{\sqrt{n}} [\sum_{j=1}^{n} g(Y_j) - n\mu_0]$$

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converges in distribution to a normal distribution with mean zero and variance σ_0^2.

Proof : Set $\tilde{g} = g - \mu_0$, thus $E_0[\tilde{g}(Y_1)] = 0$ and $E_0[\tilde{g}^2(Y_1)] < \infty$. Then, the above Theorem

follows according to the theorem on page 174 of [5].

We consider memoryless detectors with observations described by (1) which employ

nonlinearities g_k, $k = 1, 2$, that satisfy conditions similar to those presented in [3], i.e.,

$$E_{\theta k}[g_k] < \infty, \quad var_{\theta k}(g_k) < \infty \tag{4}$$

and

$$\sigma_{\theta k}^2(g_k) = var_{\theta k}[g_k(Y_1)] + 2 \sum_{j=1}^{\infty} cov_{\theta k}[g_k(Y_1)g_k(Y_{j+1})] > 0 \tag{5}$$

for all θ, where $E_{\theta k}[g_k] = \mu_{\theta k}(g_k) = \int g_k(x)f_{\theta k}(x)dx$, $f_{0k}(x)$ is the marginal pdf of $N_i^{(k)}$

(and thus of $X_i^{(k)}$ under H_0) and $f_{\theta k}(x) = f_{0k}(x - \theta)$ is the marginal pdf of $X_i^{(k)}$ under

H_1. Under these conditions and from Theorem 1 we deduce that, under H_i (for $i = 0, 1$),

the test statistics T_k (for $k = 1, 2$) are asymptotically normally-distributed with means

$n\mu_{0k}(g_k)$, $n\mu_{\theta k}(g_k)$ and variances $n\sigma_{0k}^2(g_k)$, $n\sigma_{\theta k}^2(g_k)$ for all θ. Furthermore, as $n \to \infty$ they

provide a bivariate normal distribution with the cross correlation functions $n\tilde{\rho}_0(g_1, g_2) =$

$E[(T_1 - n\mu_{01})(T_2 - n\mu_{02})]$ (under H_0) and $n\tilde{\rho}_\theta(g_1, g_2) = E[(T_1 - n\mu_{\theta 1})(T_2 - n\mu_{\theta 2})]$ (under

H_1), whose forms for the case of $g_1 = g_2$ are derived in Section IV. Throughout this paper

we assume that $\mu_{\theta k} \geq \mu_{0k}$ for $k = 1, 2$ and all $\theta > 0$.

In addition, the following regularity conditions are assumed ([3, (7)]) for $k = 1, 2$ and

all $\theta > 0$

$$\partial E_\theta[T_k]/\partial \theta|_{\theta=0} > 0 \tag{6}$$

$$\lim_{n \to \infty} \frac{\sigma_{0k}^2}{\sigma_{\theta k}^2} = 1 \tag{7}$$

$$\lim_{n \to \infty} \frac{\tilde{\rho}_0}{\tilde{\rho}_\theta} = 1 \tag{8}$$

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and

$$\partial[\int_{-\infty}^{\infty} g_k(x)f_{0k}(x-\theta)dx]/\partial\theta = \int_{-\infty}^{\infty} \partial[g_k(x)f_{0k}(x-\theta)]/\partial\theta dx \qquad (9)$$

Therefore, as $n \to \infty$, $\theta = M/\sqrt{n} \to 0$, and we define

$$\tilde{\rho}_{12} = \tilde{\rho}_0 = \tilde{\rho}_\theta$$

and

$$\sigma_k^2 = \sigma_{0k}^2 = \sigma_{\theta k}^2$$

for $k = 1, 2$.

IIB. Performance Measure and the Two-Dimensional Chernoff Bound

We use the following cost function as our performance measure

$$C(d_1, d_2; h) = \begin{cases} 0 & \text{if } d_1 = d_2 = h \\ c_1 & \text{if } d_1 \neq d_2 \\ c_2 & \text{if } d_1 = d_2 \neq h, \end{cases}$$

where $d_1, d_2, h \in \{0, 1\}$, and c_1 and c_2 are non-negative constants. Thus average cost which couples the decisions of detectors 1 and 2 is

$$E[C(d_1, d_2; k)] = \lambda E_0[C(d_1, d_2; k)] + (1 - \lambda)E_1[C(d_1, d_2; k)]$$

$$= \lambda\{c_1[P_0(1, 0) + P_0(0, 1)] + c_2(1, 1)\}$$

$$+(1 - \lambda)\{c_1[P_1(1, 0) + P_1(0, 1)] + c_2(0, 0)\}, \qquad (10)$$

where λ is the a prior probability of H_0 and P_i for $i = 0, 1$ are error probabilities under H_i. Thus $P_i(1, 0) = P_i(T_1 > n\eta_1, T_2 \leq n\eta_2), P_i(0, 1) = P_i(T_1 \leq n\eta_1, T_2 > n\eta_2)$ for $i = 0, 1$, $P_0(1, 1) = P_0(T_1 > n\eta_1, T_2 > n\eta_2)$ and $P_1(0, 0) = P_1(T_1 \leq n\eta_1, T_2 \leq n\eta_2)$, with $n\eta_k(k = 1, 2)$ the threshold employed by sensor k.

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Because of the correlation of observations across time and/or sensors, there is no closed-form expression combining all terms in the average cost. Therefore, we derive a bound on this cost and use asymptotic analysis for $n \to \infty$. First we derive the two-dimensional Chernoff bound for the general bivariate Gaussian distribution. Define

$$Y(\eta_1, \eta_2) = \begin{cases} 1 & \text{if } T_1 > n\eta_1, T_2 > n\eta_2 \\ 0 & \text{otherwise,} \end{cases}$$

where we assume that for $k = 1, 2$ $T_k \sim \mathcal{N}(n\mu_{ik}, n\sigma_k^2)$ under H_i for, $i = 0, 1$, and the pair (T_1, T_2) has bivariate Gaussian distribution with the normalized correlation coefficient $\rho_{12} = n\tilde{\rho}_{12}/(\sqrt{n}\sigma_1\sqrt{n}\sigma_2) = \tilde{\rho}_{12}/(\sigma_1\sigma_2)$. Then for $s_1 \geq 0, s_2 \geq 0$

$$e^{s_1 n\eta_1 + s_2 n\eta_2} Y(\eta_1, \eta_2) \leq e^{s_1 T_1 + s_2 T_2} \tag{11}$$

Taking the expectation of (11) under H_0 and manipulating it we obtain

$$P_0(T_1 > n\eta_1, T_2 > n\eta_2) \leq e^{-s_1 n\eta_1 - s_2 n\eta_2} E_0[e^{s_1 T_1 + s_2 T_2}]$$

$$= \sqrt{2\pi} \exp[\frac{n\sigma_1^2 s_1^2}{2} + \frac{n\sigma_2^2 s_2^2}{2} + n\rho_{12}\sigma_1\sigma_2 s_1 s_2$$

$$-s_1 n(\eta_1 - \mu_{01}) - s_2 n(\eta_2 - \mu_{02})]. \tag{12}$$

Following similar steps for the above bound we obtain for $t_1 \geq 0, t_2 \geq 0$, and $q_{i1} \geq 0, q_{i2} \geq 0$ and $r_{i1} \geq 0, r_{i2} \geq 0$

$$P_1(T_1 \leq n\eta_1, T_2 \leq n\eta_2) \leq e^{t_1 n\eta_1 + t_2 n\eta_2} E_1[e^{-t_1 T_1 - t_2 T_2}]$$

$$= \sqrt{2\pi} \exp[\frac{n\sigma_1^2 s_1^2}{2} + \frac{n\sigma_2^2 s_2^2}{2} + n\rho_{12}\sigma_1\sigma_2 s_1 s_2$$

$$+s_1 n(\eta_1 - \mu_{11}) + s_2 n(\eta_2 - \mu_{12})] \tag{13}$$

under H_1,

$$P_i(T_1 > n\eta_1, T_2 \leq n\eta_2) \leq e^{-q_{i1} n\eta_1 + q_{i2} n\eta_2} E_i[e^{q_{i1} T_1 - q_{i2} T_2}]$$

$$= \sqrt{2\pi} \exp[\frac{n\sigma_1^2 q_{i1}^2}{2} + \frac{n\sigma_2^2 q_{i2}^2}{2} - n\rho_{12}\sigma_1\sigma_2 q_{i1} q_{i2}$$

$$-q_{i1} n(\eta_1 - \mu_{i1}) + q_{i2} n(\eta_2 - \mu_{i2})] \qquad (14)$$

under $H_i, i = 0, 1$ and

$$P_i(T_1 \leq n\eta_1, T_2 > n\eta_2) \leq e^{+r_{i1} n\eta_1 - r_{i2} n\eta_2} E_i[e^{-r_{i1} T_1 + r_{i2} T_2}]$$

$$= \sqrt{2\pi} \exp[\frac{n\sigma_1^2 r_{i1}^2}{2} + \frac{n\sigma_2^2 r_{i2}^2}{2} - n\rho_{12}\sigma_1\sigma_2 r_{i1} r_{i2}$$

$$+r_{i1} n(\eta_1 - \mu_{i1}) - r_{i2} n(\eta_2 - \mu_{i2})] \qquad (15)$$

under $H_i, i = 0, 1$. The four bounds computed in equations (12)-(15) give information about the performance measure defined in (10). The positive constants s_k, t_k, r_{ik}, q_{ik} can be adjusted for setting the exact form of the bound. Since the weight (c_2) of $P_0(1, 1)$ or $P_1(0, 0)$ is greater than that (c_1) of $P_i(0, 1)$ or $P_i(1, 0)$ for $i = 0, 1$, we first minimize the bounds in (12) and (13) with respect to (s_1, s_2) and (t_1, t_2). If we minimize (12) with respect to (s_1, s_2), we have

$$P_0(T_1 > n\eta_1, T_2 > n\eta_2)$$

$$\leq \sqrt{2\pi} \exp\{-\frac{n}{2(1-\rho_{12}^2)}[\frac{(\eta_1 - \mu_{01})^2}{\sigma_1^2} + \frac{(\eta_2 - \mu_{02})^2}{\sigma_2^2}$$

$$-\frac{2\rho_{12}(\eta_1 - \mu_{01})(\eta_2 - \mu_{02})}{\sigma_1\sigma_2}]\} \qquad (16)$$

and the corresponding (s_1, s_2) are

$$\tilde{s}_1 = \frac{(\eta_1 - \mu_{01})\sigma_2 - (\eta_2 - \mu_{02})\rho_{12}\sigma_1}{(1-\rho_{12}^2)\sigma_1^2\sigma_2} \qquad (17)$$

$$\tilde{s}_2 = \frac{(\eta_2 - \mu_{02})\sigma_1 - (\eta_1 - \mu_{01})\rho_{12}\sigma_2}{(1-\rho_{12}^2)\sigma_2^2\sigma_1}, \qquad (18)$$

where $0 \leq \rho_{12}^2 \leq 1$. Similarly, if we minimize (13) with respect to (t_1, t_2), we have

$$P_1(T_1 \leq n\eta_1, T_2 \leq n\eta_2)$$

$$\leq \sqrt{2\pi} \exp\left\{-\frac{n}{2(1-\rho_{12}^2)}\left[\frac{(\mu_{11}-\eta_1)^2}{\sigma_1{}^2} + \frac{(\mu_{12}-\eta_2)^2}{\sigma_2{}^2}\right.\right.$$
$$\left.\left.-\frac{2\rho_{12}(\mu_{11}-\eta_1)(\mu_{12}-\eta_2)}{\sigma_1\sigma_2}\right]\right\} \qquad (19)$$

and the corresponding (t_1, t_2) are

$$\bar{t}_1 = \frac{(\mu_{11}-\eta_1)\sigma_2 - (\mu_{12}-\eta_2)\rho_{12}\sigma_1}{(1-\rho_{12}^2)\sigma_1{}^2\sigma_2} \qquad (20)$$

$$\bar{t}_2 = \frac{(\mu_{12}-\eta_2)\sigma_1 - (\mu_{11}-\eta_1)\rho_{12}\sigma_2}{(1-\rho_{12}^2)\sigma_2{}^2\sigma_1} \qquad (21)$$

Associating the restrictions $s_k \geq 0$ and $t_k \geq 0$ for $k = 1, 2$, we have

$$\frac{\eta_1 - \mu_{01}}{\sigma_1} > \frac{\eta_2 - \mu_{02}}{\sigma_2}\rho_{12} \qquad (22)$$

$$\frac{\eta_2 - \mu_{02}}{\sigma_2} > \frac{\eta_1 - \mu_{01}}{\sigma_1}\rho_{12} \qquad (23)$$

and

$$\frac{\mu_{11} - \eta_1}{\sigma_1} > \frac{\mu_{12} - \eta_2}{\sigma_2}\rho_{12} \qquad (24)$$

$$\frac{\mu_{12} - \eta_2}{\sigma_2} > \frac{\mu_{11} - \eta_1}{\sigma_1}\rho_{12} \qquad (25)$$

Let us now consider (14) and (15). We can minimize (14) with respect to (r_{i1}, r_{i2}), or (15) with respect to (q_{i1}, q_{i2}). However, either minimization will result in conditions contradictory to (22)-(23) or (24)-(25), if the constraint $r_{ik} \geq 0$ or $q_{ik} \geq 0$ for $i = 0, 1; k = 1, 2$ is to be satisfied. Thus we choose

$$r_{01} = \frac{\eta_1 - \mu_{01}}{\sigma_1^2}, \quad r_{02} = 0 \qquad (26)$$

$$r_{12} = \frac{\mu_{12} - \eta_1}{\sigma_2^2}, \quad r_{11} = 0 \qquad (27)$$

and

$$q_{02} = \frac{\eta_1 - \mu_{02}}{\sigma_2^2}, \quad q_{01} = 0 \qquad (28)$$

$$q_{11} = \frac{\mu_{11} - \eta_1}{\sigma_1^2}, \quad q_{12} = 0, \tag{29}$$

which give the following bounds on (14) and (15).

$$P_i(T_1 > n\eta_1, T_2 \le n\eta_2) \le \sqrt{2\pi} \exp[\frac{-n(\eta_1 - \mu_{i1})^2}{2\sigma_1^2}] \tag{30}$$

and

$$P_i(T_1 \le n\eta_1, T_2 > n\eta_2) \le \sqrt{2\pi} \exp[\frac{-n(\eta_2 - \mu_{i2})^2}{2\sigma_2^2}] \tag{31}$$

Since all parameters s_k, t_k, r_{ik} and q_{ik} for $i = 0, 1$ and $k = 1, 2$ are nonnegative, the constraints to be satisfied by η_k, μ_{ik} and σ_k are presented in (22)-(25) and

$$\mu_{0k} \le \eta_k \le \mu_{1k} \tag{32}$$

for $k = 1, 2$. In the Sections III and IV, we choose suitable thresholds $\eta_k(k = 1, 2)$ that satisfy (22)-(25) and (31).

Finally the form of the bound on the average cost is

$$
\begin{aligned}
E[C&(d_1, d_2; k)] \\
&\le \lambda\{c_1\{\sqrt{2\pi} \exp[\frac{-n(\eta_1 - \mu_{01})^2}{2\sigma_1^2}] + \sqrt{2\pi} \exp[\frac{-n(\eta_2 - \mu_{02})^2}{2\sigma_2^2}]\} \\
&\quad + c_2\sqrt{2\pi} \exp\{-\frac{n}{2(1 - \rho_{12}^2)}[\frac{(\eta_1 - \mu_{01})^2}{\sigma_1^2} + \frac{(\eta_2 - \mu_{02})^2}{\sigma_2^2} \\
&\qquad\qquad - \frac{2\rho_{12}(\eta_1 - \mu_{01})(\eta_2 - \mu_{02})}{\sigma_1\sigma_2}]\}\} \\
&\quad + (1 - \lambda)\{c_1\{\sqrt{2\pi} \exp[\frac{-n(\eta_1 - \mu_{11})^2}{2\sigma_1^2}] + \sqrt{2\pi} \exp[\frac{-n(\eta_2 - \mu_{12})^2}{2\sigma_2^2}]\} \\
&\quad + c_2\sqrt{2\pi} \exp\{-\frac{n}{2(1 - \rho_{12}^2)}[\frac{(\eta_1 - \mu_{11})^2}{\sigma_1^2} + \frac{(\eta_2 - \mu_{12})^2}{\sigma_2^2} \\
&\qquad\qquad - \frac{2\rho_{12}(\eta_1 - \mu_{11})(\eta_2 - \mu_{12})}{\sigma_1\sigma_2}]\}\} \\
&= B_d
\end{aligned}
\tag{33}
$$

III. Dependence of Observations Across Time

In this case, there is no correlation between the two sensors. The two sensors make their own decisions, according to local observations, and are coupled through a common average cost function. The optimal test is shown to be the likelihood test with a data-dependent threshold (see [1]). The computation of the optimal thresholds is complex and depends greatly on the probability distribution of the observations. Here we consider a suboptimal approach by using the Central Limit Theorem and the bounds derived in the previous section. The determination of the thresholds is easy to implement and for large sample sizes it only depends on the mean of the nonlinearity under either hypothesis.

Since the correlation coefficient ρ_{12} in Section II is zero for this case, the bound of (33) in Section II from (16), (19), (30) and (31) is

$$
\begin{aligned}
B_d = \lambda \{ & c_1 \sqrt{2\pi} \exp[\frac{-n(\eta_1 - \mu_{01})^2}{2\sigma_1^2}] + c_1 \sqrt{2\pi} \exp[\frac{-n(\eta_2 - \mu_{02})^2}{2\sigma_2^2}] \\
& + c_2 \sqrt{2\pi} \exp[\frac{-n(\eta_1 - \mu_{01})^2}{2\sigma_1^2} + \frac{-n(\eta_2 - \mu_{02})^2}{2\sigma_2^2}] \} \\
+ (1 - \lambda) \{ & c_1 \sqrt{2\pi} \exp[\frac{-n(\eta_1 - \mu_{11})^2}{2\sigma_1^2}] + c_1 \sqrt{2\pi} \exp[\frac{-n(\eta_2 - \mu_{12})^2}{2\sigma_2^2}] \\
& + c_2 \sqrt{2\pi} \exp[\frac{-n(\eta_1 - \mu_{11})^2}{2\sigma_1^2} + \frac{-n(\eta_2 - \mu_{12})^2}{2\sigma_1^2}] \}
\end{aligned}
\tag{34}
$$

where $\mu_{1k} = \mu_{\theta k}$ ($k = 1, 2$) for the weak-signal model of Section IIA. Notice that there are two parts in (34): one is the error probability under H_0, while the other is the error probability under H_1. Using for each part the fact that as $n \to \infty$

$$
a_1 e^{na_2} + b_1 e^{nb_2} \approx 2a_1 b_1 e^{(na_2 + nb_2)/2}
\tag{35}
$$

with a_1, b_1 positive constants and a_2, b_2 negative constants, we can approximate the bound

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in (34) by

$$B_d \approx \lambda A \exp\{\frac{-3n}{8}[\frac{(\eta_1 - \mu_{01})^2}{\sigma_1^2} + \frac{(\eta_2 - \mu_{02})^2}{\sigma_2^2}]\}$$

$$+(1 - \lambda)A \exp\{\frac{-3n}{8}[\frac{(\eta_1 - \mu_{\theta1})^2}{\sigma_1^2} + \frac{(\eta_2 - \mu_{\theta2})^2}{\sigma_2^2}]\}, \tag{36}$$

where A is a known constant. Since the optimal thresholds are hard to compute, we determine them in a practical way. As mentioned above, the exponents in (36) come from two types of error probabilities. These error probabilities depend on the distributions of the data and cannot be known a priori. To balance these unknown error probabilities so that we do not bias either hypothesis, we pose the condition

$$\frac{(\eta_1 - \mu_{01})^2}{\sigma_1^2} + \frac{(\eta_2 - \mu_{02})^2}{\sigma_2^2} = \frac{(\eta_1 - \mu_{\theta1})^2}{\sigma_1^2} + \frac{(\eta_2 - \mu_{\theta2})^2}{\sigma_2^2} \tag{37}$$

for all μ_{0k}, $\mu_{\theta k}$ and σ_k ($k = 1, 2$). One solution of (37) for (η_1, η_2) is

$$\eta_k = \frac{\mu_{0k} + \mu_{\theta k}}{2}, \quad k = 1, 2 \tag{38}$$

Notice that constraints (22)-(25) and (32) are satisfied automatically. Thus the bound (36) with $\eta_k(k = 1, 2)$ given by (38) is

$$B_d \approx A \exp\{\frac{-3n}{32}[\frac{(\mu_{\theta1} - \mu_{01})^2}{\sigma_1^2} + \frac{(\mu_{\theta2} - \mu_{02})^2}{\sigma_2^2}]\} \tag{39}$$

Minimizing the average cost defined by (39) is equivalent to maximizing the following exponent

$$\frac{n(\mu_{\theta1}(g_1) - \mu_{01}(g_1))^2}{\sigma_1^2(g_1)} + \frac{n(\mu_{\theta2}(g_2) - \mu_{02}(g_2))^2}{\sigma_2^2(g_2)}$$

$$= M^2\{\frac{[(\mu_{\theta1}(g_1) - \mu_{01}(g_1))/\theta]^2}{\sigma_1^2(g_1)} + \frac{[(\mu_{\theta2}(g_2) - \mu_{02}(g_2))/\theta]^2}{\sigma_2^2(g_2)}\} \tag{40}$$

Recall that the weak signal has the form $\theta = M/\sqrt{n}$, hence as $n \to \infty$, $\theta \to 0$ and (40) becomes proportional to the following performance measure

$$J(g_1, g_2) = \frac{\{\partial E_\theta[g_1]/\partial\theta\}^2_{\theta=0}}{\sigma_1^2(g_1)} + \frac{\{\partial E_\theta[g_2]/\partial\theta\}^2_{\theta=0}}{\sigma_2^2(g_2)}, \tag{41}$$

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which corresponds to the *ARE* in single-sensor detection (see [3]). Since the decisions of the two sensors are independent of each other, we can maximize each term in (41) with respect to $g_k(k = 1, 2)$. Notice that each term in (41) is invariant under the scaling of its g_k. Using an approach similar to that of [3], we have the following integral equations for $g_k(k = 1, 2)$.

$$-f'_{0k}(x)/f_{0k}(x) - \int_{-\infty}^{\infty} K_k(x, y)g_k(y)dy = g_k(x), \ k = 1, 2 \tag{42}$$

where $g_{lo,k}(x) = -f'_{0k}(x)/f_{0k}(x)$ is the locally-optimum detector (see [3]) and the integral kernel $K_k(x, y)$ has the form

$$K_k(x, y) = 2 \sum_{i=1}^{m} f_{N_{i+1}/N_1}^{(k)}(y|x) - (2m + 1)f_{0k}(y), \ k = 1, 2. \tag{43}$$

For the $\phi-$mixing noise model $m \to \infty$, while for the $m-$dependent model (see [3]) m is finite. In the simulation examples of Section V, we set $c_1 = 2$, $c_2 = 1$, and $\lambda = 1/2$, which corresponds to the criterion of minimizing error probability in single-sensor detection.

IV. A Simple Case of Dependent Data Across Time and Sensors

If the weak signal observed by the two sensors passes through the same channel, the statistics of the observations obtained locally by each sensor can be modeled as being identical. Therefore, $\mu_{01} = \mu_{02} = \mu_0$, $\mu_{\theta 1} = \mu_{\theta 2} = \mu_\theta$, and $\sigma_1 = \sigma_2 = \sigma$ In this case, we use the same detection structure for each sensor and thus the same nonlinearity $g_1(\cdot) = g_2(\cdot) = g(\cdot)$ and $\eta_1 = \eta_2 = \eta$. The bound described by (33) has the form

$$B_d = \lambda\{2\sqrt{2\pi}c_1\exp[\frac{-n(\eta-\mu_0)^2}{2\sigma^2}]+\sqrt{2\pi}c_2\exp[\frac{-n(\eta-\mu_0)^2}{(1+\rho_{12})\sigma^2}]\}$$
$$+(1-\lambda)\{2\sqrt{2\pi}c_1\exp[\frac{-n(\eta-\mu_\theta)^2}{2\sigma^2}]+\sqrt{2\pi}c_2\exp[\frac{-n(\eta-\mu_\theta)^2}{(1+\rho_{12})\sigma^2}]\} \tag{44}$$

Using the balance conditions

$$\frac{-n(\eta - \mu_0)^2}{2\sigma^2} = \frac{-n(\eta - \mu_\theta)^2}{2\sigma^2} \tag{45}$$

$$\frac{-n(\eta - \mu_0)^2}{(1 + \rho_{12})\sigma^2} = \frac{-n(\eta - \mu_\theta)^2}{(1 + \rho_{12})\sigma^2}, \tag{46}$$

we obtain one solution for η, i.e.,

$$\eta = \frac{\mu_0 + \mu_\theta}{2} \tag{47}$$

Again conditions (22)-(25) and (32) are satisfied with this threshold. Thus, the bound obtained by employing this threshold is

$$B_d = 2\sqrt{2\pi}c_1 \exp\{\frac{-n(\mu_\theta - \mu_0)^2}{8\sigma^2}\} + \sqrt{2\pi}c_2 \exp\{\frac{-n(\mu_\theta - \mu_0)^2}{4(1 + \rho_{12})\sigma^2}\} \tag{48}$$

There are two exponents in (48), namely

$$R_1(g) = \frac{n(\mu_\theta(g) - \mu_0(g))^2}{2\sigma^2(g)} = M^2\frac{[(\mu_\theta(g) - \mu_0(g))/\theta]^2}{8\sigma^2(g)}$$

and

$$R_2(g) = \frac{n(\mu_\theta(g) - \mu_0(g))^2}{(1 + \rho_{12}(g))\sigma^2(g)} = M^2\frac{[(\mu_\theta(g) - \mu_0(g))/\theta]^2}{4(1 + \rho_{12}(g))\sigma^2(g)} \tag{49}$$

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For the same reason as that of Section II, these exponents approach

$$R_1(g) = M^2 \frac{\{[\partial E_\theta[g]/\partial\theta]_{\theta=0}\}^2}{8\sigma^2(g)}$$

and

$$R_2(g) = M^2 \frac{\{[\partial E[g]/\partial\theta]_{\theta=0}\}^2}{4(1+\rho_{12}(g))\sigma^2(g)} \tag{50}$$

as $n \to \infty$ and $\theta \to 0$. The optimal choice of the nonlinearity $g(.)$ that minimizes (48) is hard to identify. However, it can be shown that, for each term R_1 or R_2, there is an optimal solution for $g(.)$. Therefore, we adopt the following practical solution: we optimize (maximize) R_1 and R_2 in (50) with respect to $g(.)$ separately, and choose the $g(.)$ that gives better performance (smaller average cost). For R_1, the same integral equation as (42) of Section III has to be solved for the optimal nonlinearity. For R_2, we can derive the integral equation satisfied by the optimal nonlinearity for following similar steps to those in [3]

Notice that for all $\alpha \neq 0$, $R_2(g) = R_2(\alpha g)$, namely R_2 is invariant under the scaling of g. Thus, maximizing R_2 with respect to g is equivalent to maximizing $(\int g f_0')^2$ and thus $-\int g f_0'$, since $-\int g f_0' = |\int g f_0'|$ because of (6), under the constraint that $(1 + \rho_{12})\sigma^2$ be equal to a constant. Let g_0 maximize R_2 under the assumption (6)

$$g_0 = arg\{\max_g H(g)\}, \tag{51}$$

where

$$H(g) = -\int g f_0' + \lambda_L(1 + \rho_{12}(g))\sigma^2(g) \tag{52}$$

with λ_L being the Lagrange multiplier. Comparing the above $H(g)$ with that in [3], we note that there is an additional term $\rho_{12}(g)\sigma^2(g)$ which we have to consider in the course

of pursuing optimal g. This term has the following form for ϕ—mixing dependence across sensors

$$\rho_{12}(g)\sigma^2(g)$$

$$= \lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} \sum_{j=1}^{n} E[(g(x_i^{(1)}) - \mu_0)(g(x_j^{(2)}) - \mu_0)]$$

$$= E[g(x_1^{(1)})g(x_1^{(2)})] + 2 \sum_{j=1}^{m} E[g(x_1^{(1)})g(x_{j+1}^{(2)})] - (2m + 1)\mu_0^2 \qquad (53)$$

where m is finite for m—dependent noise and $m \to \infty$ for ϕ—mixing noise.

Let us denote the variation of g as δg and let $J_g(\epsilon) = H(g + \epsilon \delta g)$. Then the necessary and sufficient conditions satisfied by g_0 for arbitrary δg is

$$J'_{g_0}(0) = 0 \qquad (54)$$

and

$$J''_{g_0}(0) < 0. \qquad (55)$$

Referring to equations (12) through (13) of [3] and using (53) above, we have the following necessary condition for g_0

$$J'_g(0) = \int_{-\infty}^{\infty} \{ -f_0'(x) + 2\lambda_L[g(x)f_0(x) $$

$$+ \int_{-\infty}^{\infty} g(y)(\sum_{j=1}^{m} f_j(x,y) + \tilde{f}_0(x,y))dy] \} \delta g(x) dx \qquad (56)$$

where

$$\tilde{f}_0(x,y) = (\tilde{f}_{N_1^{(1)},N_2^{(2)}}(x,y) + \tilde{f}_{N_1^{(1)},N_2^{(2)}}(y,x))/2 - 2f_0(x)f_0(y)$$

and

$$f_j(x,y) = f_{N_1^{(1)},N_{j+1}^{(1)}}(x,y) + f_{N_1^{(1)},N_{j+1}^{(1)}}(y,x)$$

$$+ \tilde{f}_{N_1^{(1)},N_{j+1}^{(2)}}(x,y) + \tilde{f}_{N_1^{(1)},N_{j+1}^{(2)}}(y,x) - 4f_0(x)f_0(y)$$

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with $f_{N_1^{(1)}, N_{j+1}^{(1)}}$ being the joint probability density function of $N_1^{(1)}$ and $N_{j+1}^{(1)}$ and $\tilde{f}_{N_1^{(1)}, N_{j+1}^{(2)}}$ the joint density function of $N_1^{(1)}$ and $N_{j+1}^{(2)}$. From (54) and (56), we deduce an integral equation similar to the one in [3] for arbitrary δg, namely

$$- f_0'(x) + 2\lambda_L[g(x)f_0(x) + \int_{-\infty}^{\infty} g(y)(\sum_{j=1}^{m} f_j(x,y) + \tilde{f}_0(x,y))dy] = 0, \qquad (57)$$

where λ_L is a scaling factor of g. From (55) we obtain

$$2\lambda_L(1 + \rho_{12}(\delta g))\sigma^2(\delta g) < 0 \qquad (58)$$

Since $\rho_{12}(.) < 1$, (47) (a sufficient condition for g_0) holds for negative λ_L. If we set $\lambda_L = -1/2$, the integral equation to be solved for g_0 is

$$- f_0'(x)/f_0(x) - \int_{-\infty}^{\infty} K_c(x,y)g(y)dy = g(x), \qquad (59)$$

where $g_{lo}(x) = -f_0'(x)/f_0(x)$ is the locally-optimum detector for this case, and $K_c(x,y)$ is the kernel of this integral equation and has a form

$$
\begin{aligned}
K_c(x,y) &= (\tilde{f}_{N_1^{(2)}/N_1^{(1)}}(y|x) + \tilde{f}_{N_1^{(1)}/N_1^{(2)}}(y|x))/2 - 2f_0(y) \\
&\quad + \sum_{j=1}^{m}[f_{N_{j+1}^{(1)}/N_1^{(1)}}(y|x) + f_{N_1^{(1)}/N_{j+1}^{(1)}}(y|x) \\
&\quad + \tilde{f}_{N_{j+1}^{(2)}/N_1^{(1)}}(y|x) + \tilde{f}_{N_{j+1}^{(1)}/N_1^{(2)}}(y|x) - 4f_0(y)] \\
&= \tilde{f}_{N_1^{(2)}/N_1^{(1)}}(y|x) - 2f_0(y) + 2\sum_{j=1}^{m}[f_{N_{j+1}^{(1)}/N_1^{(1)}}(y|x) \\
&\quad + \tilde{f}_{N_{j+1}^{(2)}/N_1^{(1)}}(y|x) - 2f_0(y)]
\end{aligned} \qquad (60)
$$

V. Simulation Results

In the Examples below, we consider m–dependent noise with $m = 100$ and sample size $n = 1000$. In Figures 1-8, we plot the receiver operating characteristic (ROC) for Gaussian (Figures 1, 2, 5 and 6) and Rayleigh (Figures 3, 4, 7 and 8) noise processes. In these figures the two types of average cost, E_0 and E_1, are normalized with respect to λ and $1 - \lambda$ respectively. Examples 1 and 2 correspond to independent data across sensors (Section III), while Examples 3 and 4 correspond to dependent data across sensors (Section IV).

Example 1: Denote by $N_i^{(k)}$ the noise of sensor k at time instant i for $k = 1, 2; i = 1, \cdots, n$. Consider the noise processes characterized by

$$N_1^{(k)} = V_1^{(k)}$$

$$N_i^{(k)} = \rho_k N_{i-1}^{(k)} + \sqrt{1 - \rho_k^2} V_i^{(k)}$$

where $V_i^{(k)} \sim \mathcal{N}(0, \sigma_k^2)$ for $i = 2, \cdots, n; k = 1, 2$ are $i.i.d.$ Gaussian random variables, and $V_i^{(1)}$, $V_i^{(2)}$ are independent of each other for all $i = 1, \cdots, n$. Thus, for $k = 1, 2$

$$f_{0k}(N_i^{(k)}) = \exp[-(N_i^{(k)})^2/2\sigma_k^2]/(2\pi\sigma_k^2)^{1/2}, \quad i = 1, \cdots, n$$

and

$$f^{(k)}(N_1^{(k)}, N_{i+1}^{(k)})$$
$$= \frac{1}{2\pi(1 - \rho_k^{2i})^{1/2}\sigma_k^2} \exp\{\frac{-1}{2\sigma_k^2(1 - \rho_k^{2i})}[(N_1^{(k)})^2 + (N_{i+1}^{(k)})^2$$
$$-2\rho_k^i N_1^{(k)} N_{i+1}^{(k)}]\}$$

where $\rho_k^i = E[N_1^{(k)} N_{i+1}^{(k)}]/\sigma_k^2$ for $j = i, \cdots, n$ and $k = 1, 2$. Under hypothesis H_0 the observations $X_i^{(k)} = N_i^{(k)}$; under hypothesis H_1, $X_i^{(k)} = \theta + N_i^{(k)}$. The particular case

with $\rho_1 = 0.9$, $\rho_2 = 0.7$, $\sigma_1 = 3$, $\sigma_2 = 5$ is illustrated in Figures 1 and 2 for different η_1. In both figures solid lines represent the ROC of the detector using the nonlinearities g_k ($k = 1, 2$) and dot-dash lines represent the one using the likelihood-ratio detector for *i.i.d.* data.

Fig. 1 $\eta_1 = -5.7$

Fig. 2 $\eta_1 = 5.4$
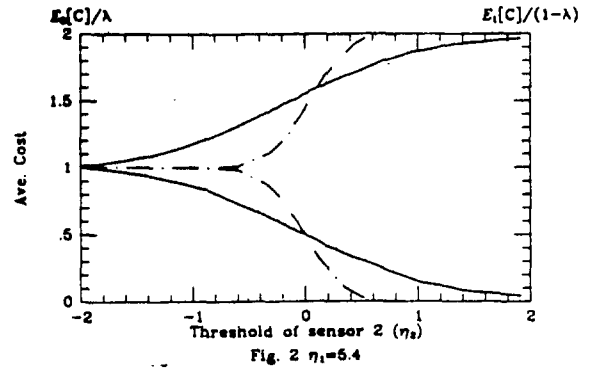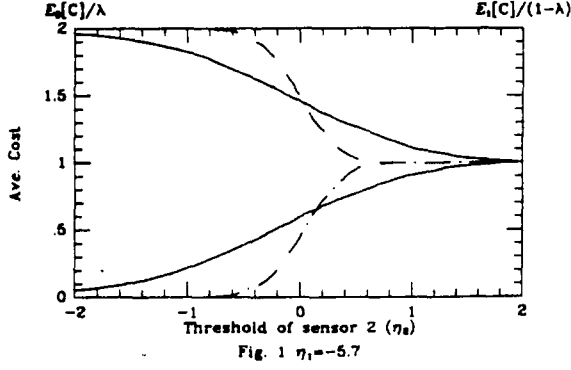
Example 2: Let

$$Z_i^{(k)} = \sqrt{(X_i^{(k)})^2 + (Y_i^{(k)})^2}, \quad k = 1, 2; \ i = 1, \cdots, n$$

where $X_i^{(k)}$ is generated by the same model as $N_i^{(k)}$ in Example 1 and $Y_i^{(k)}$ is described by

$$Y_1^{(k)} = W_1^{(k)}$$

$$Y_i^{(k)} = \rho_k Y_{i-1}^{(k)} + \sqrt{1 - \rho_k^2} W_i^{(k)}, \quad k = 1, 2; \ i = 2, \cdots, n$$

with $W_i^{(k)} \sim \mathcal{N}(0, \sigma_k^2)$ and $W_i^{(1)}, W_i^{(2)}$ independent of each other. Then $Z_i^{(k)}$ has a Rayleigh distribution given by

$$f_{0k}(Z_1^{(k)}) = \frac{(Z_1^{(k)})^2}{\sigma_k^2} \exp[-\frac{(Z_1^{(k)})^2}{2\sigma_k^2}], \quad k = 1, 2$$

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and a second-order joint pdf given by

$$f^{(k)}(Z_1^{(k)}, Z_{i+1}^{(k)})$$

$$= \frac{Z_1^{(k)} Z_{i+1}^{(k)}}{(1 - \rho_k^{2j})\sigma_1^2 \sigma_2^2} \exp\left\{ -\frac{1}{2(1 - \rho_k^{2j})} \left[\frac{(Z_1^{(k)})^2}{\sigma_1^2} + \frac{(Z_{i+1}^{(k)})^2}{\sigma_2^2} \right] \right\}$$

$$\cdot I_0\left[\frac{\rho_k^j Z_1^{(k)} Z_{i+1}^{(k)}}{(1 - \rho_k^{2j})\sigma_1 \sigma_2} \right], \quad i = 1, \cdots, n; k = 1, 2$$

Now consider the observations of the two sensors. Under H_0 the observations $Z_i^{(k)}$ are Rayleigh distributed. Under H_1 the observations $Z_i^{(k)}$ has the following form

$$Z_i^{(k)} = \sqrt{(X_i^{(k)} + \theta)^2 + (Y_i^{(k)})^2}, \quad i = 1, \cdots, n; k = 1, 2$$

which has a Rician distribution for nonzero θ, and asymptotically a Rayleigh distribution as $\theta \to 0$. In fact, these observations correspond to the envelope of a weak signal in narrowband noise. Although it cannot be modeled directly by an additive noise model, we use it to test the detection schemes derived in Sections III and IV. In this case, the locally-optimum detector in (42) and (59) has, for $k = 1, 2$, the form $g_{lo,k}(x) = -f'_{0k}(x)/(x f_{0k}(x))$ for Section III and $g_{lo}(x) = -f'_0(x)/(x f_0(x))$ for Section IV (see [6]). Figures 3 and 4 show simulation results for this case with parameters $(\rho_1, \rho_2, \sigma_1 \text{ and } \sigma_2)$ being the same as those in Example 1. In these figures solid lines and dot-dash lines have the same meaning as those in Example 1.


Fig. 3 $\eta_1 = -9900$

Fig. 4 $\eta_1 = 54$
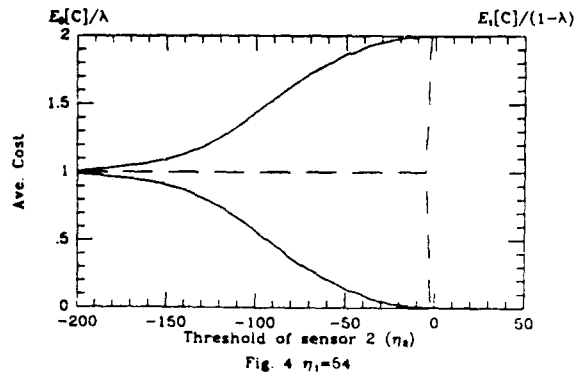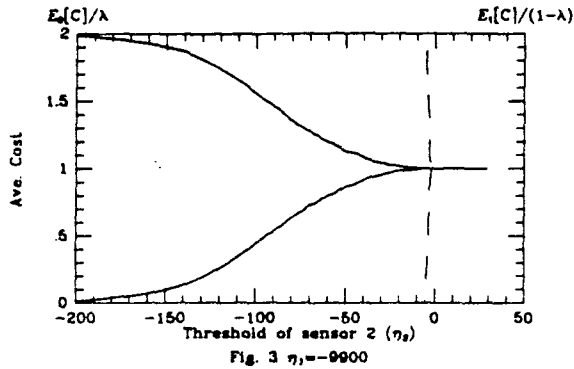
Example 3: Consider again the noise model given in Example 1. Suppose that $\sigma_1 = \sigma_2 = \sigma$, $\rho_1 = \rho_2 = \rho$ and the sequences $V_i^{(1)}$, $V_i^{(2)}$ are correlated through the correlation coefficient ρ_V as

$\rho_V = E[V_i^{(1)}V_i^{(2)}]/\sigma^2$. Thus, for $j = 1, \cdots, n$

$$E[N_1^{(1)}N_{j+1}^{(2)}] = \rho^j \rho_V \sigma^2$$

Figures 5 and 6 show simulation results for the case of $\rho = 0.7$, $\sigma = 5$ In Figure 5, $\rho_V = 0.4$, while in Figure 6 $\rho_V = -0.4$. Let g and g_c denote the nonlinearities obtained by optimizing the exponents R_1 and R_2, respectively. In these figures solid lines represent the .ROC of the detector using the nonlinearity g, while dash lines and dot-dash lines represent the ones using the nonlinearity g_c and the locally-optimum detector for $i.i.d.$ data, respectively.

Fig. 5 $\rho_{V}=0.4$ Fig. 6 $\rho_{V}=-0.4$
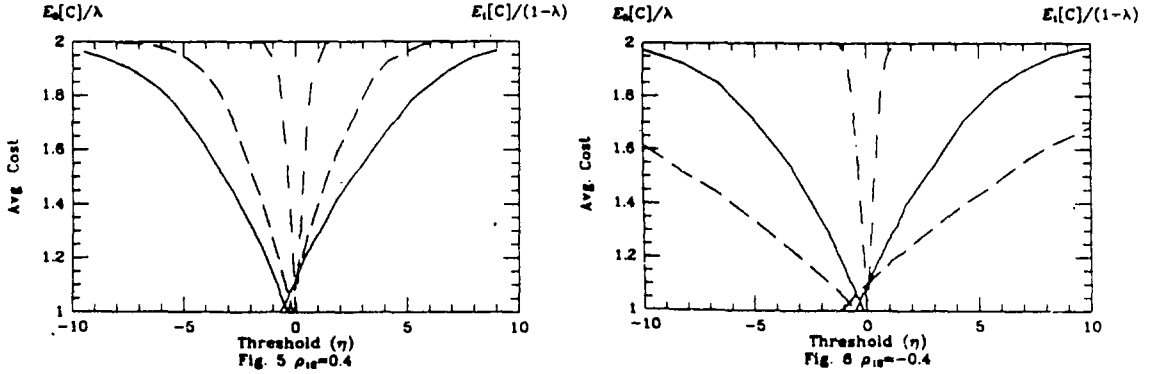
Example 4: Consider again the Rayleigh noise processes given in Example 2. Let the parameters (σ_k and ρ_k for $k = 1, 2$) be the same as those in Example 3. Furthermore, define $\rho_W = E[W_i^{(1)}W_i^{(2)}]/\sigma^2$ and let $\rho_W = \rho_V$ for $i = 1, \cdots, n$. Figures 7 and 8 show the results for the cases with $\rho_V = \rho_W = 0.4$ and $\rho_V = \rho_W = 0.9$ respectively. In both figures,

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solid, dash and dot-dash lines have the same meaning as those in Example 3.

Fig. 7 $\rho_{12} = 0.4$

Fig. 8 $\rho_{12} = 0.9$
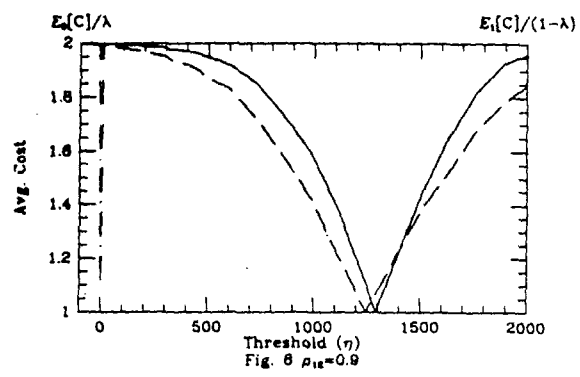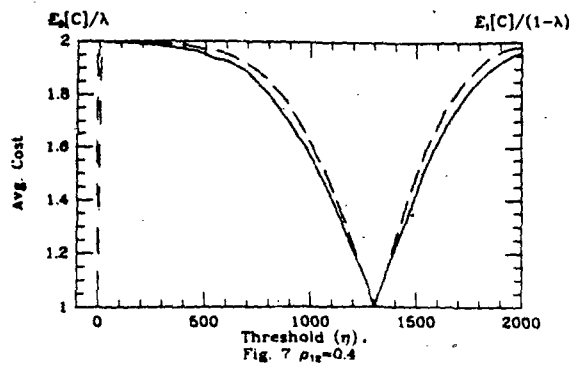
VI. Conclusions

In this paper we extended the memoryless detection of a known weak signal in dependent noise to the case of distributed two-sensor detection from correlated sensor observations. The correlation of noise across time and/or sensors is characterized by m-dependent or ϕ-mixing models. We devised two-dimensional Chernoff bounds on the average error probability of the two detectors and from those obtained performance measures resembling (although distinctly different) the asymptotic relative efficiency (ARE) for the two-sensor problem. Optimization of this performance measure led to linear integral equations whose solutions provided the optimal memoryless nonlinearities used by the sensors. Our results are applicable to both cases of symmetric and asymmetric correlated noise. The simulation results obtained suggest that, regarding the average error probability of the two sensors, employing memoryless nonlinearities that take into account the correlation in the samples of the two detectors, is always better than using the locally optimal nonlinearity that ignores the dependence between samples. Takin into consideration the correlation of observations across sensors improves the performance when the overall correlation is negative for Gaussian observations or heavy positive for Rayleigh observations.

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